differentiate chain rule
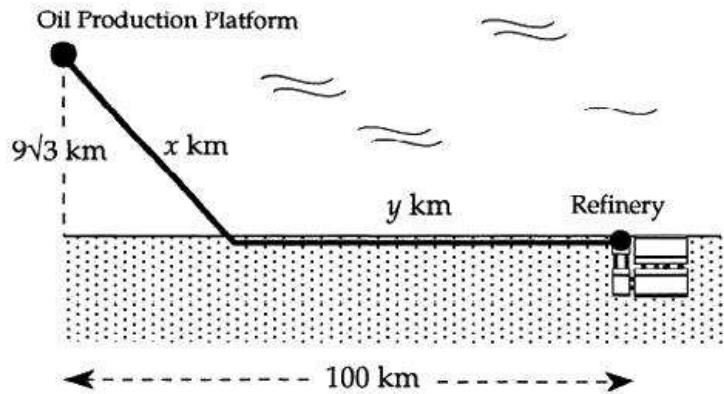
1. An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.

The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

(a) Show that the total cost of this pipeline is £$C(x)$ million where

$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}.$$

(3)

(b) Show that $x = 18$ gives a minimum cost for this pipeline. Find this minimum cost and the corresponding total length of the pipeline. (7)

2. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$. 3

3. (a) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. 2

(b) Hence write down $\displaystyle\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}} \, dx$. 1

4. Given that $f(x) = 5(7 - 2x)^3$, find the value of $f'(4)$. 4

5. Differentiate $2x^{\frac{3}{2}} + \sin^2 x$ with respect to x. 4

6. Find the derivative, with respect to x, of $\dfrac{1}{x^3} + \cos 3x$. 4

7. Linktown Church is considering designs for a logo for their Parish magazine. The 'C' is part of a circle and the centre of the circle is the mid-point of the vertical arm of the 'L'. Since the 'L' is clearly smaller than the 'C', the designer wishes to ensure that the total length of the arms of the 'L' is as long as possible.
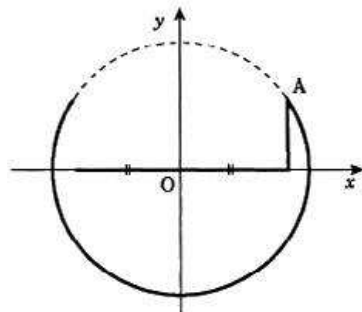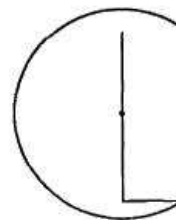
The designer decides to call the point where the 'L' and 'C' meet A and chooses to draw co-ordinate axes so that A is in the first quadrant. With axes as shown, the equation of the circle is $x^2 + y^2 = 20$.

(a) If A has co-ordinates (x,y), show that the total length T of the arms of the 'L' is given by $T = 2x + \sqrt{20 - x^2}$. (1)

(b) Show that for a stationary value of T, x satisfies the equation

$$x = 2\sqrt{20 - x^2}.$$ (5)

(c) By squaring both sides, solve this equation.
Hence find the greatest length of the arms of the 'L'. (3)

8. If $f(x) = \cos^2 x - \dfrac{2}{3x^2}$, find $f'(x)$. **4**

9. Differentiate $4\sqrt{x} + 3\cos 2x$ with respect to x. **4**

10. Find $\dfrac{dy}{dx}$ given that $y = \sqrt{1 + \cos x}$. **3**

11. Given $f(x) = (\sin x + 1)^2$, find the exact value of $f'(\frac{\pi}{6})$. **3**

[END OF QUESTIONS]